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„Statistical analysis of the results in brief”

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***Outline***

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1. Statistics definition.
2. Hypothesis testing.
3. Type of variables.
4. Normal distribution (average, SD, median, modal).
5. Software for statistical analysis.
6. Parametric and nonparametric test examples and practical application.



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***Statistics***

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- Statistics - it is the method of getting the knowledge about tested parameters of the whole population (people, animals, plants, group of materials) on the basis of properly selected (REPRESENTATIVE!!!) sample.
- In practice: with statistics we are searching the answer to the question:” Do the samples differ from each other?”
- Finally… there is no 100% sure answer. Statistics analysis forecasts the results with some level of uncertainty (usually 5%).

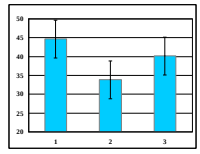


Fig. 1. Measured values of different samples



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***Statistics***

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There are three levels of lies:

- forecasting
- diplomatic message
- statistics

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***Hypothesis testing***

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The general idea of hypothesis testing involves:

- Making an initial assumption ($H_0$ – null hypothesis, $H_1$ – alternative hypothesis, determining the significance level (0.05)).
- (Significance level – the probability of $H_0$ rejection)
- Collecting evidence (data, variables).
- Based on the available evidence (data), deciding whether to reject or not reject ($H_0$) the initial assumption.

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***Hypothesis testing - examples***

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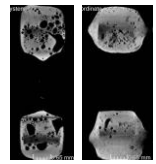
Proper formulating of the problem/research hypothesis.

Examples:

- $H_0$ – there is no difference in the voids content between the joints fabricated with convection reflow or VPS technology.
- $H_1$- there is the difference in the voids content between the joints fabricated with convection reflow or VPS technology.



But: The void total volume in the joint is important but also the size of the single void.

Fig. 2. Void formation in the solder joints

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## *Hypothesis testing - examples*

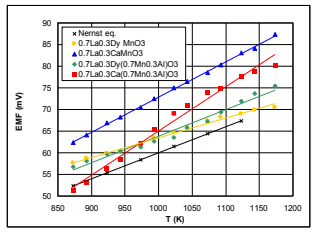


Fig. 3. EMF values versus temperature

- $H_0$ – the plots of the electromotive force changes for the SOFC samples with different electrode materials are the same as theoretical curve obtained from Nernst equation.
- $H_1$ – the plots of the electromotive force changes for the SOFC changes differ form theoretical curve obtained from Nernst equation.



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## *Hypothesis testing*

- We always assume the null hypothesis is true and that is no difference between the samples.

But:

- If we reject the null hypothesis, we do not prove that the alternative hypothesis is true.
- If we do not reject the null hypothesis, we do not prove that the null hypothesis is true.

*„Presumption of innocence" - one is considered innocent unless proven guilty.*

- Two types of errors :"Type I error" and "Type II error
  - Type I error: The null hypothesis is rejected when it is true.
  - Type II error: The null hypothesis is not rejected when it is false.

*And so one and so one……..*



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## *Hypothesis testing*

In statistics, there are two ways to determine whether the evidence is likely or unlikely given the initial assumption:

- "critical value approach" (favored in many of the older textbooks) → Excel, manually calculated statistics.
- "P-value approach" (what is used most often in research, journal articles, and statistical software) →advanced statistics software.

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## *Variables (data)*

- A **variable** is an object, event, idea, feeling, time period, or any other type of category you are trying to measure. There are two types of variables-independent and dependent.
- **Independent variables** are variables that are manipulated or are changed by researchers and whose effects are measured and compared. The other name for independent variables is Predictor(s).
- The other variable(s) are also considered the **dependent variable**(s). The dependent variables refer to that type of variable that measures the affect of the independent variable(s) on the test units.

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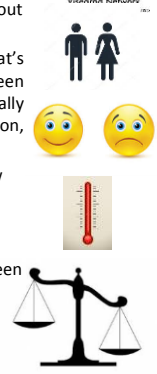
## *Variables scales*

- **Nominal Scales** - are used for labeling variables, without any quantitative value (gender, colors, place of living)
- **Ordinal Scale** - the order of the values is what's important and significant, but the differences between each one is not really known. Ordinal scales are typically measures of non-numeric concepts like satisfaction, happiness, discomfort, etc.
- **Interval Scales** - are numeric scales in which we know not only the order, but also the exact differences between the values (Celsius temperature - the difference between each value is the same.)
- **Ratio Scales** - tell about the order - exact value between units, AND they also have an absolute zero–which allows for a wide range of both descriptive and inferential statistics to be applied.



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## *Scales - comparison*

| Provides: | Nominal | Ordinal | Interval | Ratio |
|---|---|---|---|---|
| The "order" of values is known | | ✔ | ✔ | ✔ |
| "Counts," aka "Frequency of Distribution" | ✔ | ✔ | ✔ | ✔ |
| Mode | ✔ | ✔ | ✔ | ✔ |
| Median | | ✔ | ✔ | ✔ |
| Mean | | | ✔ | ✔ |
| Can quantify the difference between each value | | | ✔ | ✔ |
| Can add or subtract values | | | ✔ | ✔ |
| Can multiple and divide values | | | | ✔ |
| Has "true zero" | | | | ✔ |

http://www.mymarketresearchmethods.com/types-of-data-nominal-ordinal-interval-ratio/

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## *Normal distribution (Gaussian)*

The Normal Distribution has:

- mean = median = mode (central tendency)
- symmetry about the center ⟶ Parametrical tests
- 50% of values less than the mean and 50% greater than the mean

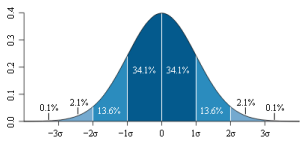


Fig. 3. Normal distribution
https://pl.wikipedia.org/wiki/Rozk%C5%82ad_normalny

σ - Standard deviation (s )

68% of values are within 1 standard deviation of the mean

95% of values are within 2 standard deviations of the mean

99,7% of values are within 3 standard deviations of the mean



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## *Normal distribution (Gaussian)*

- How to check if your data the "normally distributed"?
  - Kolmogorov-Smirnov test for normality
  - Shapiro–Wilk test
- The other way is to calculate the average, SD, median, min and max and evaluate the results.
- If SD is high and median is slightly different form the average there is the risk that the distribution in not normal.
- If the range (min and max values) is wide it is recommended to use non-parametric tests.
- Pay attention on the **outliers** (the observations that are distant from other observations). If the measured values are not included in the range of ±2SD from the average it should be rejected or replaced!!



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## *Means*

- Arithmetic mean - the sum of the numbers divided by how many numbers are being averaged. Arithmetic average should be calculated only when adding up the values makes sense !!

$$\overline{X} = \frac{\sum X}{N}$$

- **Weighted mean** - if the numbers of next measurements are different

e.g.. The researcher measured of the shear strength for the SAC solder joints. The results came from 3 substrates with mounted resistors (1 substrate - 10 resistors, 2 – substrate -15 resistors, 3 – substrate 5 resistors).

$$\overline{X}_w = \frac{10 \cdot \overline{X} + 15 \cdot \overline{X} + 5 \cdot \overline{X}}{10+15+5}$$

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## *Variance, standard deviation*

- The **variance** ($\sigma^2$ – whole population, $s^2$ – sample population) is a measure of **how far each value in the data set is from the mean.**

$$s^2 = \frac{\sum_{i=1}^{n}\left(x_i - \overline{X}\right)^2}{n-1}$$

- Standard deviation is calculated as the square root of the variance.

$$s = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \overline{X}\right)^2}{n-1}}$$

$x_i$ is the $i^{th}$ observation from a sample of the population,

x-bar is the sample mean,

n (sample size) -1 is degrees of freedom,

Σ is the summation



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## *Median, Mode*

- The **median** is the value separating the higher half of a data sample, a population, or a probability distribution, from the lower half. For a data set, it may be thought of as the "middle" value (the median is the most resistant statistics).

  Examples:

  1, 3, 3, 6, 7, 8, 9 — median = 6

  1, 2, 3, 4, 5, 6, 8, 9 — median = 4.5
- The **modal** it is the value that is most likely (the most frequent)

  Examples:

  1, 3, 6, 6, 6, 6, 7, 7, 12, 12, 17

  modal = 6

  (the most frequent size of the single void)



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## *Statistics software*

- Free statistical software is a practical alternative to commercial packages.
- These packages come from a variety of sources, including governments, nongovernmental organizations (NGOs) like UNESCO, and universities, and are also developed by individuals.
  - PSPP
  - EasyReg
  - Openstat
  - Online
    - http://www.socscistatistics.com/tests/Default.aspx
- Commercial
  - Excel
  - Statistica (Statsoft –Dell)

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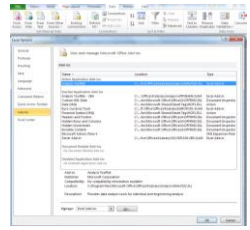
## *Excel- data analysis*


- If it is not available – install Analysis ToolPak
- Click the File tab, click Options, and then click the Add-Ins category
- In the **Add-Ins** box, check the **Analysis ToolPak** check box, and then click **OK**

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## *Parametric and nonparametric tests*

| Parametric tests<br>(normal distibution) | Nonparametric tests<br>(any distribution including normal) |
|---|---|
| 1. **Differences testing between independent groups** | |
| t-test<br>ANOVA | U-test (Mann-Whitney)<br>Kruskall-Wallis test by ranks |
| **2. Differences testing between dependent groups** | |
| t-test | Wilcoxon signed-rank test |
| **3. Correlation between variables** | |
| Pearson correlation coefficient | R Spearman test |
| Regression | |

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## *Parametric tests*

- **Parametric test** is one that makes assumptions about the parameters (defining properties) of the population distribution(s) from which one's data are drawn.
- A parametric test is more able to reject of $H_0$.
- One- and two-tailed tests
  - A two-tailed test is appropriate if the estimated value may be more than or less than the reference value.
  - A one-tailed test is appropriate if the estimated value may depart from the reference value in only one direction (just differ).

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## *Nonparametric tests*

- If the distribution is not normal and the number of the sample is not high.
- **Nonparametric tests** do not rely on any distribution.
- They can be applied even if parametric conditions of validity are not met.



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## *t-test (parametric, independent groups)*

- The *t-test* is any statistical hypothesis test in which the test statistic follows a Student's t-distribution under the null hypothesis.
- The *t-test* can be used, for example, to determine if two groups of data are significantly different from each other.
- Several groups can not be compared with each other by performing *t-test* several times.
- The idea is to compare the mean and standard deviation of one group of subjects with the predetermined value.

$$t = \frac{\overline{X_1} + \overline{X_2}}{\sqrt{\left(\frac{(N_1-1)s_1^2 + (N_2-1)s_2^2}{N_1+N_2-2}\right)\left(\frac{1}{N_1}+\frac{1}{N_2}\right)}}$$



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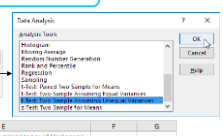
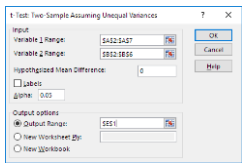
## *t-test example (Excel)*

Comparison of the number of voids in SAC alloys – $H_0$ says that SAC305 and SAC307 have the same number of the voids.

Number of voids in the joints

| SAC305 | SAC307 |
|---|---|
| 26 | 23 |
| 25 | 30 |
| 43 | 18 |
| 34 | 25 |
| 18 | 28 |
| 52 | |





| t-Test: Two-Sample Assuming Unequal Variances | | |
|---|---|---|
| | Variable 1 | Variable 2 |
| Mean | 33 | 24.8 |
| Variance | 160 | 21.7 |
| Observations | 6 | 5 |
| Hypothesized Mean Difference | 0 | |
| df | 7 | |
| t Stat | 1.47260514 | |
| P(T<=t) one-tail | 0.092170202 | |
| t Critical one-tail | 1.894578605 | |
| P(T<=t) two-tail | 0.184340405 | |
| t Critical two-tail | 2.364624252 | |

http://www.excel-easy.com/examples/anova.html

- If $t_{stat} < -t_{Critical}$ and $t_{Stat} > t_{Critical}$ (in two-tail test) we reject the null hypothesis.
- This is not the case, -2.365 < 1.473 < 2.365. Therefore, we do not reject the null hypothesis.

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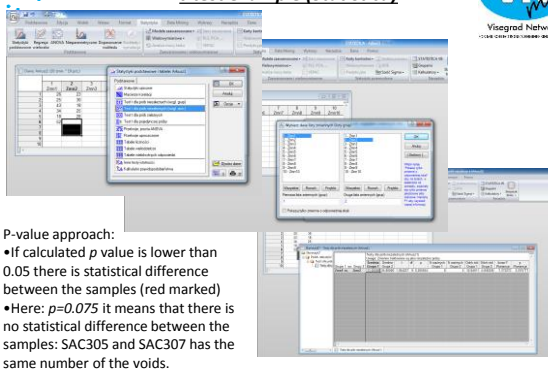
## *t-test example (Statistica)*



P-value approach:
•If calculated *p* value is lower than 0.05 there is statistical difference between the samples (red marked)
•Here: *p=0.075* it means that there is no statistical difference between the samples: SAC305 and SAC307 has the same number of the voids.

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## *Anova (parametric,* independent groups*)*

- Is one of the most popular and most commonly used statistical analyzes.
- More precisely - ANOVA it is the group of analyzes used to examine the influence of factors (independent variables) on the dependent variable.
- It is more advanced tool than *t-test.*
- The idea of variance analysis is to check whether certain independent variables (factors) influence the level of the dependent variable (measured values).
- Analysis of variance is the ratio of the variance that we calculated between the studied groups and the average variance that we have we observed inside the groups.

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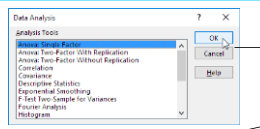
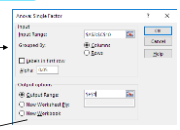
## *Anova example (Excel)*

Comparison of the shear strength values of the solder joints - $H_0$ says that that shear strength of the different solder joints doesn't differ

Shear strenth values

| SAC305 | SAC307 | mSAC |
|---|---|---|
| 42 | 69 | 35 |
| 53 | 54 | 40 |
| 49 | 58 | 53 |
| 53 | 64 | 42 |
| 43 | 64 | 50 |
| 44 | 55 | 39 |
| 45 | 56 | 55 |
| 52 | | 39 |
| 54 | | 40 |



Anova: Single Factor

SUMMARY

| Groups | Count | Sum | Average | Variance |
|---|---|---|---|---|
| Column 1 | 9 | 435 | 48.33333 | 23.5 |
| Column 2 | 7 | 420 | 60 | 32.33333 |
| Column 3 | 9 | 393 | 43.66667 | 50.5 |

ANOVA

| Source of Variation | SS | df | MS | F | P-value | F crit |
|---|---|---|---|---|---|---|
| Between Groups | 1085.84 | 2 | 542.92 | 15.19623 | 7.16E-05 | 3.443357 |
| Within Groups | 786 | 22 | 35.72727 | | | |
| Total | 1871.84 | 24 | | | | |

- Conclusion: if *F > F crit*, we reject the null hypothesis.
- This is the case, 15.196 > 3.443. Therefore, we reject the null hypothesis. The means of the three samples are not all equal.

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http://www.excel-easy.com/examples/t-test.html

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## *Anova example (Statistica)*

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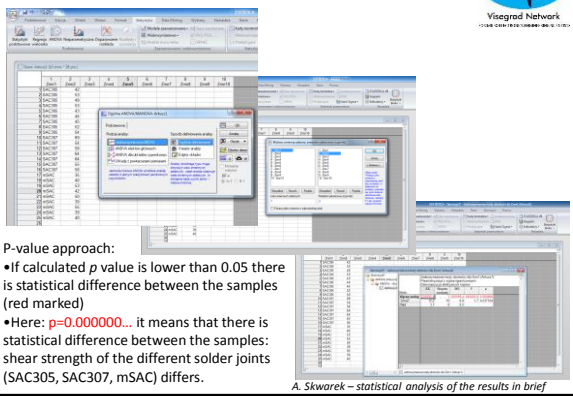

P-value approach:
•If calculated *p* value is lower than 0.05 there is statistical difference between the samples (red marked)
•Here: p=0.000000... it means that there is statistical difference between the samples: shear strength of the different solder joints (SAC305, SAC307, mSAC) differs.





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## *U-test (Mann-Whitney) (nonparametric,* **independent groups***)*

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- The *U-test* is one of the most popular alternatives for the *t-test* for independent trials.
- The dependent variable must be measured on an ordinal scale (it may also be measured on a quantitative scale).
- The use of the *U-test* does not require the group parallelism, normal distribution or homogeneous variances. This makes it widely applicable.
- The *U-test* is about ranking.
- The null hypothesis asserts that the medians of the two samples are identical.

$$U = NM + \frac{N(N+1)}{2} - \sum_{X_i} Rank(x_i)$$



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## *Ranking*

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- The ordinal number of a value in a list arranged in a specified order

| Value |
|---|
| 57 |
| 74 |
| 43 |
| 78 |
| 61 |
| 64 |
| 58 |

Arrange the data in ascending order →

| Value |
|---|
| 43 |
| 57 |
| 58 |
| 61 |
| 64 |
| 74 |
| 78 |

Give ranks →

| Rank |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |



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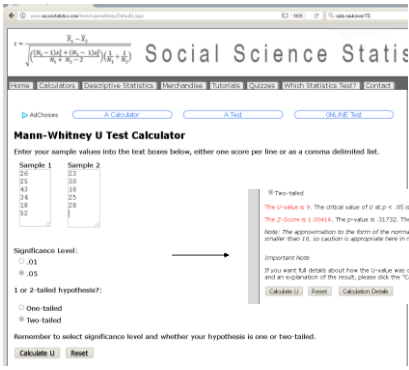
## *U-test example (Socscistatistics)*




Comparison of the number of voids in SAC alloys – $H_0$ says that SAC305 and SAC307 have the same median value of the voids number.



http://www.socscistatistics.com/tests/mannwhitney/



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## *Kruskall-Wallis test (nonparametric,* **independent groups)**

- The Kruskal-Wallis test is a non-parametric test, which does not assume that the data come from a distribution that can be completely described by two parameters, mean and standard deviation (normally distributed).
- Alternative to the one-factor ANOVA test for **independent measures**.
- Like most non-parametric tests, you perform it on ranked data, so you convert the measurement observations to their ranks in the overall data set: the smallest value gets a rank of 1, the next smallest gets a rank of 2, and so on.
- You lose information when you substitute ranks for the original values, which can make this a somewhat less powerful test than a one-way Anova.

$$H = \frac{12}{N(N+1)} \cdot \frac{\sum T^2}{n} - 3(N+1)$$





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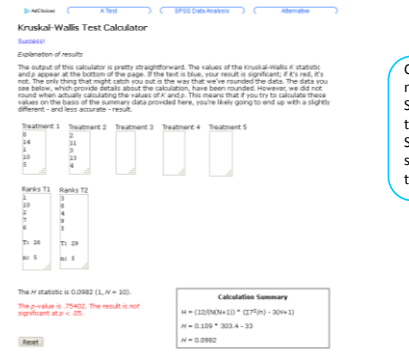
## *Kruskall-Wallis test by ranks example (Socscistatistics)*




Comparison of the number of voids in SAC alloys – $H_0$ says that SAC305 and SAC307 have the same median value of the voids number.



http://www.socscistatistics.com/tests/kruskal/Default.aspx

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## ***Wilcoxon signed-rank test (nonparametric,* dependent groups)**

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- The Wilcoxon test is a nonparametric test designed to evaluate the difference between two treatments or conditions where the samples are correlated (**dependent measures**).
- In particular, it is suitable for evaluating the data from a repeated-measures design when t-test cannot be performed.
- So, for example, it might be used to evaluate the data from an experiment that looks at the reading ability of children before and after intensive training.

The parametric alternative for ***Wilcoxon test for dependant mesurments is t-test***

$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$

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## ***Wilcoxon signed-rank test example (Socscistatistics)***

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Comparison of the number of Sn whisker before and after thermal shocks – $H_0$ says that there is no difference in Sn whisker number before and after thermal shocks.

- Wilcoxon test requires more than 20 measured values.
- Depends on the population size either *Z* value, either *W* value is compared

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http://www.socscistatistics.com/tests/signedranks/Default.aspx

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## ***Regression (correlation, parametric)***

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- The main idea of regression is prediction, forecasting data for a certain variable based on other variables.
- In regression it is assumed that the increase of one variable (predictor, predictors) is accompanied by an increase or decrease on the second variable.
- If the regression function is a linear function, i.e. y = bx + a – it is linear regression (y- dependent variable, x - independent variable, b - regression coefficient, a –absolute term).
- Determination coefficient $R^2$ - is a statistic that will give some information about the goodness of fit of a model.
- Correlation coefficient R - square root of the determination coefficient!

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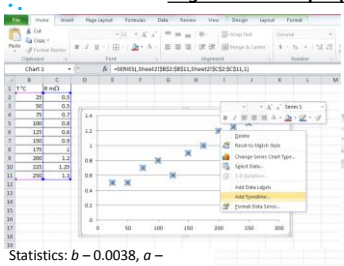
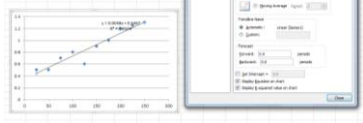
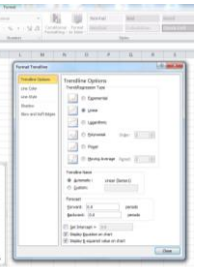
## Regression example (Excel)



The changes of the resistance vs. temperature of the structure with nano-silver joints

- Statistics: *b* – 0.0038, *a* – 0.3467 and $R^2$ – 0.9104 (R – 0.9103!!)
- To compare 2 regression coefficients → *t-test*
- There is positive linear relationship between the resistance and temperature increase.





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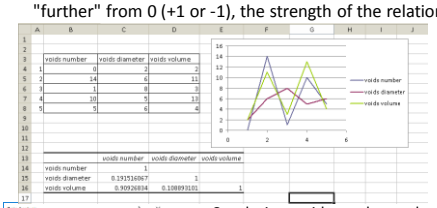
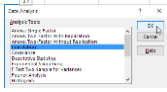
## Pearson coefficient (correlation, parametric)

- Correlation means the relationship between variables.
- The Pearson correlation coefficient is used to measure the strength of a linear association between two variables.
- It is defined as a value in intervals from -1 to 1. The coefficient is "further" from 0 (+1 or -1), the strength of the relationship is greater.



Voids number in the solder joint is correlated with their diameter and their volume.



- Conclusion: voids number and volume are correlated (0.91).
- Voids number and diameter are not correlated (0.19).
- Voids diameter and volume are not correlated (0.11) .

*A. Skwarek – statistical analysis of the results in brief*

http://www.excel-easy.com

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## R Spearman test (correlation, nonparametric)

- Spearman's Rho measures the strength and direction of the relationship between two variables.
- Requirements
  - Scale of measurement must be ordinal (or interval, ratio).
  - **Data must be in the form of matched pairs.**
  - The association must be monotonic (i.e., variables increase in value together, or one increases while the other decreases).

$$r_s = 1 - \frac{6\sum D^2}{N^3 - N}$$

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## *R Spearman test example (Socscistatistics)*

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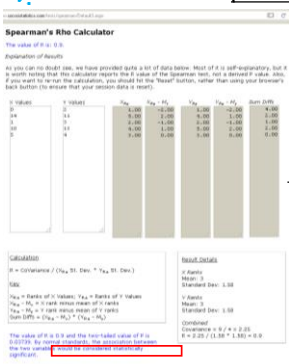
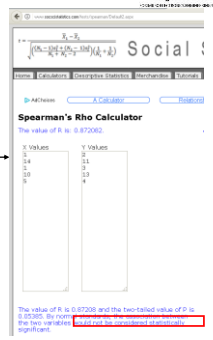



Small change in only one value

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http://www.socscistatistics.com/tests/spearman/Default.aspx



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## *Exponential smoothing (Excel)*

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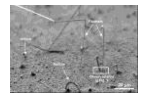
Exponential smoothing is used to smooth out irregularities (peaks and valleys) to easily recognize trends.

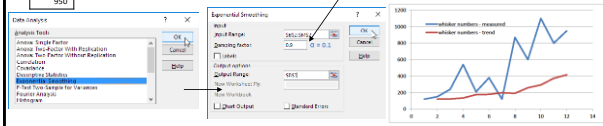
Number of whiskers

| |
|---|
| 150 |
| 240 |
| 540 |
| 210 |
| 380 |
| 120 |
| 870 |
| 400 |
| 1100 |
| 700 |
| 950 |

Click in the Damping factor box and type 0.9.

Smaller damping factors also mean that your smoothed values are closer to the actual data points than larger damping factors.

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http://www.excel-easy.com



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## *Social Science Statistics*

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The web site offers free resources for students and researchers working with statistics

SOCIAL SCIENCE STATISTICS

http://www.socscistatistics.com/Default.aspx